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Total Least-Squares Criteria in Parameter Identification for Flight Flutter Testing

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An investigation of parameter identification of time series models for linear dynamic structural systems using the least squares and total least-squares criteria is presented. Excitation and response time-domain data are used to parameterize autoregressive, moving average models. The method, or criteria, which is used to solve a set of overdetermined, linear algebraic equations developed from the time-domain data affects the solution. A commonly used criteria, least squares, introduces the possibility of significant bias error in the system parameters and leads to bias errors in the modal parameter estimates when measurement errors are present. An alternative criteria, total least squares, provides an approach that appears to significantly reduce the bias error in the parameter estimates. These methods are applied to a simple, simulated system and then to flight flutter test data with particular emphasis on accurate modal damping estimates.

Nomenclature

$[A]$	= data matrix
$[A']$	= contaminated data matrix
$[A', b']$	= augmented data matrix
$[\hat{A}, \hat{b}]$	= estimated data matrix
$\|[A]\|_F$	= Frobenius norm, $\sqrt{\sum_{i=1}^{m} \sum_{j=1}^{n} \|a_{ij}\|^2}$
a_n	= autoregressive coefficients
$\{b\}$	= observation vector
$\{b'\}$	= contaminated observation vector
$\{\hat{b}\}$	= estimated observation vector
b_n	= moving average coefficients
$\|\{b\}\|_2$	= 2 norm, $\sqrt{\sum_{i=1}^{m} \|b_i\|^2}$
$[E]$	= noise sequence
$\{e\}$	= noise sequence
f_d	= damped natural frequency
N	= number of data points
rms	= root mean square, $\sqrt{(1/N) \sum_{i=1}^{N} x_i^2}$
$[S]$	= singular value matrix, see Eq. (16)
t	= time
$[U]$	= left singular matrix, Eq. (16)
$u(t)$	= discrete excitation value
$[V]$	= left singular matrix, Eq. (16)
$[v]$	= right singular vector
$x(t)$	= discrete response value
Δt	= sampling interval
ζ	= damping ratio
$\{\theta\}$	= parameter vector
$\{\hat{\theta}\}$	= estimated parameter vector
$\{\hat{\theta}_{LS}\}$	= estimated parameter vector, least squares
$\{\hat{\theta}_{TLS}\}$	= estimated parameter vector, total least squares
Σ	= singular values
σ	= standard deviation, $\sqrt{[1/(N-1)] \sum_{i=1}^{N} (x_i - \bar{x})^2}$

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Introduction

THE use of time-domain models to represent dynamic systems has been the subject of an increasing number of research efforts over the past 20 years. This article is concerned with the use of time series difference equation models for dynamic structural systems. The models are created with the specific goal of using vibration response data to estimate the frequency and damping of the primary vibration modes of the structure. This research effort was motivated by the desire to generate estimates of modal characteristics from subcritical flight flutter test data using short data records and rapid data processing techniques. Accurate estimates of modal damping are particularly important in this application since they are often used as a indicator for flutter onset. This information must also be available in nearly real time so that decisions which influence the continuation of a flight flutter test can be made. The ability of the flutter test engineer to acquire accurate damping estimates is a current research issue since much of the earlier research in this type of digital signal processing has been conducted in the area of spectral estimation, where the primary interest is to accurately identify the frequencies of signals.

Most current flight flutter techniques are based on well-established dynamic system identification methods, and in the past have been almost exclusively frequency domain based. Multiple blocks of vibration response data are transformed from the time domain into the frequency domain where they are ensemble averaged. The resulting power spectral density or transfer function is used with curve-fitting techniques to estimate modal frequency and damping. Usually there is an attempt to reduce the influence of measurement noise in the signals by collecting many blocks of data, but this extends the amount of time that the aircraft must remain at test conditions and increases the cost of the test.

Time domain system identification provides an alternative to frequency domain methods, offering some possibilities for improved capabilities for this application. This may be the case when attempting to identify systems with closely spaced modes and when multiple output signals are available. Additionally, while frequency domain methods require the estimation of modes from curve fits of frequency domain information, a process often requiring subjective decisions, time-domain techniques allow for the direct calculation of frequency and damping directly from model parameters. Finally, in certain situations time series models can be constructed with relatively short time records.

Consider a structure subject to an external excitation to be a simple system where an input, the excitation, is applied to the system and the resulting output is the system response. Time histories of the excitation and response can be used to estimate a model for the system. In almost all practical systems the processes used to measure and record the excitation and response introduce noise into the measured signals. The challenge presented to most dynamic system modeling techniques is to construct an accurate model in the presence of this noise or measured data uncertainty.

In the current study the systems are modeled as time-invariant, linear difference equations referred to as autoregressive, moving average (ARMA) models. The specific intent of this article is to discuss the implementation of parameter estimation techniques that improve the accuracy of the system identification process. Past efforts have shown that these models can provide very good estimates of natural frequency and damping in the absence of significant measurement noise.[1-4] It is recognized that often these estimates contain both bias and precision errors, and the source and reduction of the bias error in the estimates are the primary concerns of this article.

Time Series Models of a Dynamic System

Four of the major steps involved in the system identification process addressed in this article are listed. Although each step contains a number of assumptions worthy of detailed discussion, this article is primarily concerned with the third step, the identification of the time series model parameters: 1) measure the excitation and response signal data, 2) select an appropriate form for the time series model, 3) identify the time series model parameters, and 4) use the parameters to find system characteristics. Though the third step is of primary concern, the influence of the other steps is briefly outlined in the following:

1) Excitation methods for flutter testing are often classified as either random or sinusoidal, dependent upon their time-domain character, and they can have a marked influence on the methods used for data analysis. Because of the emphasis placed upon rapid data collection in this work only random excitation methods are considered. For the current study the fact that both the excitation and response signals contain some uncertainty is particularly relevant. When conducting studies with data from simulated systems, the type and level of noise can be controlled. However, when attempting to build models of real systems from experimental data, many of the sources contributing to uncertainty are unknown.

2) Two common types of time series models are the autoregressive (AR) and ARMA models. The AR model can be shown to exactly represent multiple damped sinusoids and is appropriate for finite degree-of-freedom free-response data.[5] An AR difference equation expresses the response of a system at a time $x(t)$ as a function of previous response values $x(t - i\Delta t)$, and a finite set of system parameters a_i:

$$x(t) = a_1x(t - \Delta t) + a_2x(t - 2\Delta t) + \cdots + a_nx(t - n\Delta t) \quad (1)$$

The ARMA model is based on both the excitation and response of a system and it is an approximation for forced response. An ARMA difference equation sets the response of the system as a function of previous values of the excitation and response $u(t - i\Delta t)$ and $x(t - i\Delta t)$ of the system and a set of system parameters a_i, b_i:

$$x(t) = a_1x(t - \Delta t) + a_2x(t - 2\Delta t) + \cdots + a_nx(t - n\Delta t) + b_1u(t - \Delta t) + \cdots + b_nu(t - n\Delta t) \quad (2)$$

The number of terms included in each of these difference equations is an issue of concern and is related to the order of the system (i.e., the number of modes present in a specific frequency range) and causality. If the order of the system is unknown prior to parameter identification, additional problems are introduced. In the case of flight flutter testing, the engineer often can make a reasonable estimate of the number of structural modes within a particular frequency range.

3) The third step in the system identification process is forming and solving a set of linear algebraic equations for the unknown parameters in either the AR and ARMA models. For the AR example, Eq. (1) can be written for $m + 1$ different instants in time. The unknowns in these $m + 1$ equations are the model parameters and the system of equations can be expressed as

$$\begin{aligned} x(t_i) &= a_1x(t_{i-1}) + a_2x(t_{i-2}) + \cdots + a_nx(t_{i-n}) \\ x(t_{i+1}) &= a_1x(t_i) + a_2x(t_{i-1}) + \cdots + a_nx(t_{i-n+1}) \\ &\vdots \\ x(t_{i+m}) &= a_1x(t_{i+m-1}) + a_2x(t_{i+m-2}) + \cdots + a_nx(t_{i+m-n}) \end{aligned} \quad (3)$$

The number of equations and number of unknowns influence the methods that can be used to solve for the unknowns. As will be discussed, least squares and total least squares are two criteria that can be used to solve a system of overdetermined equations.

4) The final step in this identification process is determining the system characteristics from the time-domain model parameters. This is accomplished with a simple transformation of the time-domain difference equation into the z domain and forming the corresponding discrete time transfer function. The poles of the transfer function provide the natural frequency and damping for the system. Thus, to accurately estimate modal frequency and damping, it is necessary to accurately estimate the parameters of the difference equation.

Least-Squares and Total Least-Squares Criteria

Under certain conditions, solving for the parameters in a linear model is a simple and highly accurate process. Either AR or ARMA difference equations can be written for multiple instances in time, and form a system of linear, algebraic equations. The set of difference equations [Eq. (1) or (2)] can be written as

$$[A]\{\boldsymbol{\theta}\} = \{\boldsymbol{b}\} \quad (4)$$

If one considers an AR representation of a finite order, linear system undergoing free response, an exactly determined set of linear algebraic equations can be formed and the set has a unique solution that exactly defines the values of system parameters.

In practice, both the excitation and response signals are contaminated with noise, the order of the system is unknown, and solving for the parameters is not a simple process. A widely used approach is to write an overdetermined set of difference equations and since the solution is approximate, it is based on some criteria. The following discusses two of these criteria.

Least Squares

Perhaps the most common solution criteria for overdetermined systems of linear equations is least squares (LS). Considering Eq. 4 in the absence of measurement noise, $[A]$ is acted upon by $\{\boldsymbol{\theta}\}$ to produce the exact $\{\boldsymbol{b}\}$. If the observation vector alone is contamination by noise or errors $\{e\}$ then one needs to introduce the measured observation vector $\{\boldsymbol{b}'\}$. As a result of noise in the observations, Eq. (4) can also be expressed as

$$[A]\{\boldsymbol{\theta}\} = \{\boldsymbol{b}'\} - \{e\} \quad (5)$$

Since both the vectors $\{\boldsymbol{\theta}\}$ and $\{\boldsymbol{e}\}$ are unknown it is impossible to exactly determine the parameter vector from the measured observation vector using Eq. (5). Instead the parameter estimate is based on the measured observation vector and in a least-squares estimate one attempts to solve Eq. (6):

$$[A]\{\boldsymbol{\theta}\} = \{\boldsymbol{b}'\} \tag{6}$$

The least-squares parameter estimate is equivalent to minimizing the sum of the square of the difference between $\{\boldsymbol{b}'\}$ and $\{\hat{\boldsymbol{b}}\}$. The estimated observation vector is generated with $\{\hat{\boldsymbol{\theta}}\}$ and $[A]$ as shown in Eq. (7):

$$[A]\{\hat{\boldsymbol{\theta}}\} = \{\hat{\boldsymbol{b}}\} \tag{7}$$

The quantity to be minimized can be written in either of the forms:

$$\min(\Sigma|[A]\{\hat{\boldsymbol{\theta}}\} - \{\boldsymbol{b}'\}|^2) = \min(\Sigma|\{\hat{\boldsymbol{b}}\} - \{\boldsymbol{b}'\}|^2) \tag{8}$$

The estimate of the parameter vector resulting from the application of the LS criteria is

$$\{\hat{\boldsymbol{\theta}}_{LS}\} = ([A]^T[A])^{-1}[A]^T\{\boldsymbol{b}'\} \tag{9}$$

Although the least-squares criteria is widely used in solving linear systems, it can be viewed as being based on the assumption that only the observation vector is contaminated with noise. This estimate is said to be unbiased if the difference between the actual parameter vector and the estimated parameter vector approaches zero as the number of equations (i.e., length of the data record) increases. There are two conditions for obtaining an unbiased least-squares estimate: 1) $[A]$ must be statistically independent of $\{\boldsymbol{e}\}$, and 2) the expected value (mean) of the noise must be zero.[6] With both AR and ARMA difference equations, the data matrix contains many of the same elements as $\{\boldsymbol{b}\}$. As a result, the data matrix is not independent of $\{\boldsymbol{e}\}$, and at least one of the conditions for an unbiased least-squares solution is not satisfied.

A bias error has been observed in many of the parameter identification methods based upon least-squares estimates.[1-4] One common method of reducing this bias error is to use a highly overspecified model (i.e., assume the model order to be significantly larger than the actual system order), but this often results in the problem of sorting system and computational modes.[1] A second method is to employ singular value decomposition (SVD) to find a lower rank approximation to the data matrix, but this requires a similar type of sorting and modification of the singular values. A third method of reducing this bias is to use an alternative to the least-squares criteria.

Total Least Squares

The total least-squares (TLS) criteria was developed as a method in which the parameter estimates are generated from a system of equations where it is assumed that both the data matrix and the observation vector are contaminated with noise. Comparatively, the LS and the TLS solve the following systems of equations:

$$\text{Least squares: } [A]\{\boldsymbol{\theta}\} = \{\boldsymbol{b}\} + \{\boldsymbol{e}\} \tag{10}$$

$$\text{Total least squares: } [[A] + [E]]\{\boldsymbol{\theta}\} = \{\boldsymbol{b}\} + \{\boldsymbol{e}\} \tag{11}$$

A characteristic of the TLS estimate is that the parameter vector can be shown to be consistent and unbiased under less restrictive conditions than the LS solution.[7]

An effective method of illustrating the total least-squares solution is to compare it directly with the least-squares solution. Based on the assumption that $[A]$ contains no errors, the least-squares solution finds a vector $\{\hat{\boldsymbol{\theta}}_{LS}\}$ to minimize the sum of the squared, or 2-norm, of the residual vector:

$$\min\|\{\hat{\boldsymbol{b}}\} - \{\boldsymbol{b}'\}\|_2 \tag{12}$$

The TLS approach assumes errors contaminate both the data matrix and the observation vector. The parameter estimate is found by seeking to minimize[8]

$$\min\|[A', \boldsymbol{b}'] - [\hat{A}\ \hat{\boldsymbol{b}}]\|_F \tag{13}$$

The matrix $[A', \boldsymbol{b}']$ is an augmented data matrix created by combining the data matrix and the observation vector, and $[\hat{A}\ \hat{\boldsymbol{b}}]$ corresponds to the predicted matrix from the TLS solution. $\|A\|_F$ is the Frobenius norm that is a measure of the size of a $m \times n$ matrix just as the 2-norm measures the length of a vector.[9]

The total least-squares solution is found by reformulating Eq. 14 as a homogeneous, linear system as shown in Eq. (15):

$$[A']\{\boldsymbol{\theta}\} = \{\boldsymbol{b}'\} \tag{14}$$

$$[A', \boldsymbol{b}']\begin{bmatrix} \boldsymbol{\theta} \\ -1 \end{bmatrix} = 0 \tag{15}$$

It can be shown that the solution to a homogeneous linear system is found in the null subspace of the augmented data matrix.[9] Although a number of methods can be used to divide the augmented data matrix into a system subspace and a null subspace, singular value decomposition is commonly used:

$$[A', \boldsymbol{b}'] = [U][S][V]^T \tag{16}$$

The matrices resulting from the SVD are partitioned into system and null subspaces:

$$[A', \boldsymbol{b}'] = [U_{\text{signal}} U_{\text{null}}]\begin{bmatrix} \Sigma_{\text{signal}} & 0 \\ 0 & \Sigma_{\text{null}} \end{bmatrix}\begin{bmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{bmatrix}^T \tag{17}$$

Although detailed development is beyond the scope of this article, Golub and Van Loan[9] show the TLS parameter estimate is based on the columns of $[V]$, which correspond to the singular values in the null subspace as shown in Eq. (17)[7]:

$$\{\hat{\boldsymbol{\theta}}_{TLS}\} = -[V_{12}][V_{22}]^{-1} \tag{18}$$

The most straightforward TLS solution occurs when the null subspace is of rank = 1 and a unique parameter estimate can be found directly. The solution to this system is the single vector that forms a basis for the null subspace. This vector corresponds to the smallest singular value, and therefore is the last column of the $[V]$ matrix. The estimate of the parameter vector is written as

$$\{\hat{\boldsymbol{\theta}}_{TLS}\} = -\frac{\{v_{(1,n+1)}, v_{(2,n+1)}, \ldots, v_{(n,n+1)}\}}{v_{(n+1,n+1)}} \tag{19}$$

If the rank of the null subspace is greater than one, multiple estimates of the parameter vector will exist. In this situation, one approach is to accept all of the solutions as estimates of the parameter vector and then perform some type of averaging. A second approach is to combine all of the estimates into a single parameter estimate. If the order of the system is known, a linear combination of the estimates can be used to find the minimum norm TLS solution, which is the approach taken in this study. Finally, it is important to note that the order of the difference equation used to develop the original data matrix must be at least as large as the order of the system being modeled for the null subspace to exist.

Parameter Identification, Order Overspecification, and Backward Methods

The preceding discussion focused on the solution of a system of overdetermined, linear algebraic equations. As mentioned, the solution of sets of equations of this type is only one of the steps that take place in a number of time-domain parameter identification methods. The work presented in this article is based upon the extension of the reduced backward method (RBM) approach to time-domain parameter identification.[10] RBM was developed to provide an automated procedure for time-domain parameter identification with particular emphasis on short time record analysis. It assumes no prior knowledge of the system characteristics (i.e., number of modes in a specified bandwidth). In RBM noise bias reduction was achieved through order overspecification. Automated pole sorting, required because of the model order overspecification, was accomplished by reformulating the ARMA models in the backwards time sense. Stable (damped) modes in the forward time sense become unstable when the time history data is reordered so that time is decreasing. It is then possible to automatically discriminate between the system poles and the computational poles introduced because of order overspecification.

Even with large degrees of order overspecification RBM produced biased parameter estimates. Though acceptable for certain applications, this was acceptable when applied to flutter test data processing.[11] This was particularly true of the bias error associated with the damping estimates because of the role that damping estimates play in the flutter point prediction. The RBM resulted in damping estimates consistently less than those produced by frequency domain techniques. Even though these estimates were biased by the presence of measurement noise in the data, the RBM approach provided certain advantages. The process was automated, it typically required significantly less data (i.e., shorter time records that could be a big advantage in the flight test environment), and it was well suited for differentiating between very closely spaced modes.

The research presented in the current paper was an attempt to exploit the advantages associated with both the RBM and the TLS criteria. By applying TLS to the parameter identification approach used in RBM it was hoped to reduce the required levels of overspecification that would reduce both the data requirements and computational times. If this could be accomplished along with a reduction in the bias error, the combined approach could provide synergism useful in the flight flutter test environment.

Applications

The following describes the application of the total least-squares criteria in the parameter identification procedure for both simulated and actual systems. ARMA model parameters were identified using random, forced response data from a numerically simulated, dynamic system and from an experimental flight flutter test. The numerically simulated system provided a controlled situation under which the relative characteristics of the LS and TLS criteria could be compared within the context of the same parameter identification procedure. The flight test data provided a most rigorous and realistic test environment for the identification methods.

Simulated System Response

The behavior of a dynamic system was numerically simulated to examine the accuracy of system identification procedures using least squares and total least squares. Although both the bias and precision error were calculated, the reduction of bias error produced with the RBM was the primary objective.

The simulated system response data was generated using an ARMA model with a given set of parameters and a prescribed, random input time series. The model parameters were chosen so that a single mode system was approximated with a damped natural frequency of 7 Hz and a damping ratio of 0.02. The ARMA representation of the system is shown in Eq. (20) for a sample frequency of 40 Hz:

$$x(t) = 1.2526x(t - \Delta t) - 0.9654x(t - 2\Delta t) + 2u(t - \Delta t) - 1.2526u(t - 2\Delta t) \qquad (20)$$

A time series response was generated from this ARMA model with a normally distributed, random white noise excitation. Both the excitation and response time series records were contaminated with independent, normally distributed random noise. Time series records containing 256 data points with a uniform time interval were used. These relatively short time series records would represent a total sampling time of approximately 6.4 s. A sample excitation, response, noise sequence, and contaminated response are shown in Fig. 1. The magnitude of the random noise excitation was selected to keep the ratio of the rms of the response to the rms of the noise equal for both the excitation and response records. The signal-to-noise ratio in Fig. 1 is 4.0.

The simulated system was used to evaluate three issues. In the first study, a second-order ARMA model was used with the LS and TLS methods. Because the model was not overspecified, there were no extraneous computational modes and no mode sorting issues, and the influence of order overspecification on the bias errors was not present. Four different system identification approaches were considered by using both the LS and TLS criteria to solve forward and backward formulations of the ARMA equations.

In each of the four cases all 256 data points were used to create a highly overdetermined set of linear algebraic equations. The LS parameter estimates were obtained using a SVD to solve the overdetermined set of equations, and the TLS solutions were achieved using the approach outlined earlier. The identification process was repeated 10 times with 10 different sets of noise-contaminated excitation and response signals (S:N = 4:1). Note that this is particularly noisy data, but it can be characteristic of the noise level encountered in certain types of experiments. This is particularly true for flight flutter testing where there are many possible noise sources.

Since order overspecification was not used, the characteristic equation developed from the ARMA model was second order and it resulted in a single pole in the discrete time or z domain. The position of this pole is used to determine the modal frequency and damping. Figure 2 shows the pole positions and Table 1 shows the frequency and damping resulting from forward time estimates using both LS and TLS criteria. As shown in numerous previous studies the LS estimates are significantly biased. Because each pole estimate is biased toward the origin of the unit circle in the z plane (Fig. 2), the damping is overestimated (Table 1). The standard deviation of the sample is also included in Table 1 and provides an estimate of the precision error associated with each parameter. Using the TLS

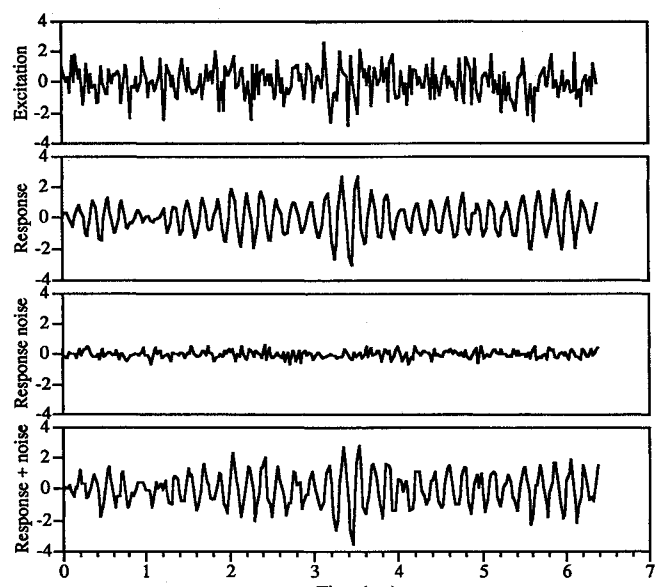

Fig. 1 Forced response of simulated dynamic system.

Table 1 System identification results with forward model

	Least squares		Total least squares	
	Frequency	Damping	Frequency	Damping
	7.256	0.0661	6.989	0.0209
	7.208	0.0645	7.031	0.0188
	7.181	0.0622	6.960	0.0182
	7.227	0.0624	6.956	0.0196
	7.268	0.0663	6.974	0.0219
	7.208	0.0652	6.992	0.0274
	7.234	0.0798	7.065	0.0210
	7.235	0.0669	7.012	0.0218
	7.187	0.0579	6.992	0.0193
	7.281	0.0653	7.013	0.0196
Mean	7.228	0.0657	6.998	0.0209
Sample σ	0.033	0.0056	0.033	0.0026

Table 2 System identification results with backward model

	Least squares		Total least squares	
	Frequency	Damping	Frequency	Damping
	7.180	−0.0275	6.989	0.0209
	7.217	−0.0288	7.031	0.0188
	7.134	−0.0267	6.960	0.0182
	7.157	−0.0324	6.956	0.0196
	7.158	−0.0266	6.974	0.0219
	7.226	−0.0336	6.992	0.0274
	7.269	−0.0312	7.065	0.0210
	7.223	−0.0337	7.012	0.0218
	7.174	−0.0278	6.992	0.0193
	7.237	−0.0395	7.013	0.0196
Mean	7.197	−0.0308	6.998	0.0209
Sample σ	0.043	0.0041	0.033	0.0026

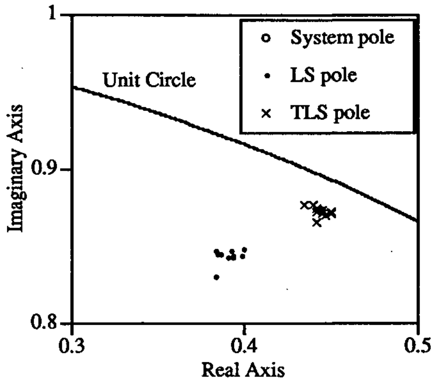

Fig. 2 Estimated pole positions (z plane) from forward model.

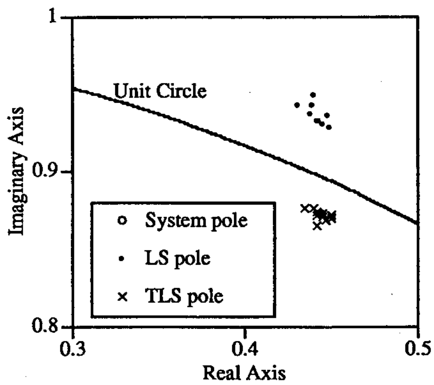

Fig. 3 Estimated pole positions (z plane) from backward model.

criteria the bias errors in the frequency and damping estimates have been significantly reduced. The pole positions are still biased slightly toward the origin of the unit circle and a small overestimate of the modal damping results. Though the value of the standard deviation of the sample for the 10 estimates is also reduced in TLS, it represents a greater percentage of the mean damping value.

Figure 3 and Table 2 present comparable results using the same simulated data with a backward time approach. In the case of the LS criteria the pole estimates were again biased toward the origin of the unit circle. When the pole locations were then transformed to their appropriate forward time location, they actually ended up outside the unit circle (Fig. 3) and had negative damping values (Table 2). It will be illustrated that if the signal-to-noise ratio is increased, the mean LS damping estimate becomes less negative, and in the forward time sense the pole would eventually move back inside the unit circle and have positive damping. The underestimation of damping using the backward time approach exists whenever measurement noise is present.

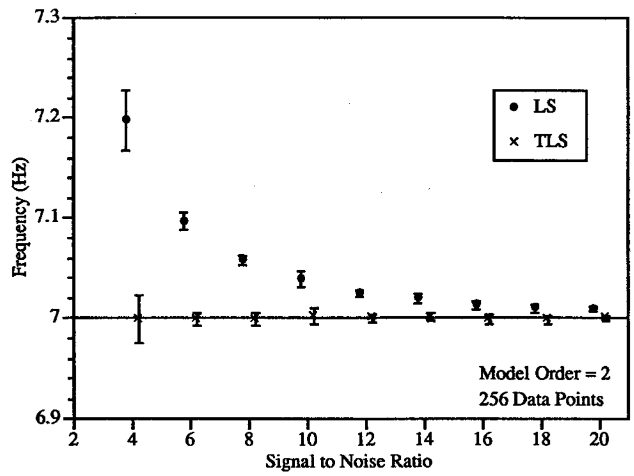

Fig. 4 Averaged modal frequency estimates of numerically simulated system.

Table 2 also presents the estimates using the TLS criteria. Note that the TLS criteria yields the exact same parameter estimates, for both the forward or backward time models as would be expected since the augmented data matrix [Eq. (15)], is the same regardless of the direction of time. The mean values for both the frequency and damping estimates are good, particularly considering that the signal-to-noise ratio is only 4:1. The standard deviation of the sample for the damping estimates is not insignificant. Although TLS has reduced the bias error, the presence of noise still effects the precision of the individual parameter estimates. Averaging multiple estimates (i.e., reducing the standard deviation of the mean) then becomes an effective way of reducing the precision error and improving the parameter estimates.

Figures 4 and 5 provide additional insight to the influence on the frequency and damping estimates from both LS and TLS solution criteria at different signal-to-noise ratios. The error bands indicated for each estimate represent the 95% confidence limits assuming a normal distribution and small sample size statistics. At each noise level 10 sets of contaminated excitation and response data were used with a second-order, backward, ARMA model to generate 10 estimates of frequency and damping. The 10 estimates were averaged to produce one estimate at a number of signal-to-noise levels. The bias error in the LS frequency and damping estimates are shown as the difference between the true value of the parameter and the mean of the 10 samples. As the signal quality is improved the influence of the measurement noise on the LS estimate is reduced, but the TLS estimates provide improved estimates over a rather wide range of signal-to-noise ratios. It is important to note that the bias error in the LS solution consistently causes the damping to be underestimated.

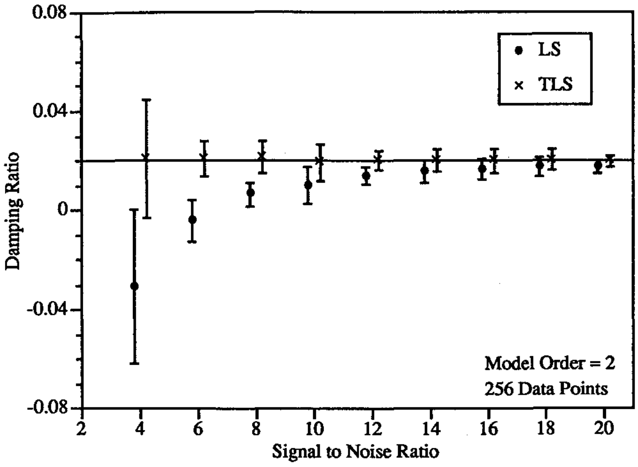

Fig. 5 Averaged damping ratio estimates of numerically simulated system.

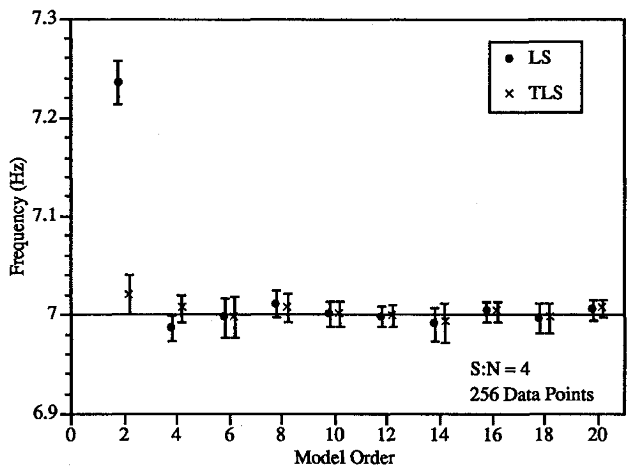

Fig. 6 Averaged modal frequency estimates of numerically simulated system.

The use of overspecified models in the identification process has been an effective method of improving the accuracy of parameter estimates. The additional or computational modes in an overspecified model provide an outlet for the noise in the signal that would otherwise cause error in the parameter estimates. For this study, a signal-to-noise ratio of 4:1 was chosen and 10 sets of contaminated excitation and response data were generated. Backward ARMA models with orders ranging from 2 to 20 were used for system identification with both the LS and TLS criteria. It is clear from Fig. 6 that even low levels of overspecification can be effective in improving the LS frequency estimates. With regard to the damping estimates (Fig. 7), the bias error that is seen in the LS estimate is reduced as the overspecification level is increased. It can be seen that the LS estimate consistently underestimates the damping when compared to the TLS estimate. These figures do not indicate that there are dramatic improvements in the TLS estimates with order overspecification.

Although significantly simpler than an actual dynamic system, some observations can be made using the results presented with this simulated system. Most importantly, the least squares criteria clearly leads to biased parameter estimates, and in a backward model, this bias causes the damping estimate to be consistently underestimated. Although overspecification can be effective in reducing this bias error, it persists even at high levels of overspecification. The TLS criteria offers an effective method of reducing the bias error in parameter identification. It has also been shown that the precision error is primarily related to the signal-to-noise ratio, and the ability of overspecification to reduce precision error is somewhat limited. Averaging multiple estimates provides a method by which the precision error in the parameter estimate can be reduced. Finally, it is the damping estimate that is most significantly affected by the bias error seen in the LS solutions.

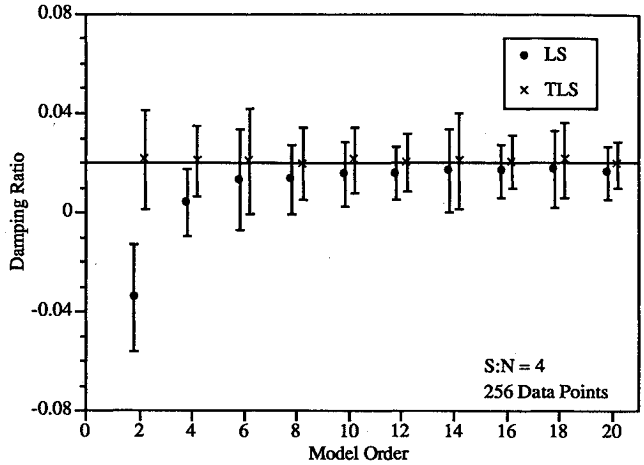

Fig. 7 Averaged damping ratio estimates of numerically simulated system.

Flutter Flight Test Data

Flight flutter test data were used in the second demonstration of this modal parameter estimation technique. It provided a realistic environment in which the technique could be evaluated. The test data used in this study were generated by the U.S. Air Force in a series of F-16 flight flutter tests. It was made available to NASA for an evaluation of the structural excitation techniques and was used in the current study to evaluate the capability of a TLS-based parameter estimation method. Unfortunately, because the data results from the response of a real structure in a very complex environment, there are no exact frequency and damping values to provide comparisons as was done with the simulated data. Instead, frequency and damping estimates have been generated using both frequency domain and time domain identification techniques and these results are compared. Because the frequency domain techniques are based on an established estimation method, they provide the best comparison available for this data. This approach also allowed for the comparison of computational and engineering effort associated with each approach and these are to be discussed in a qualitative manner later. Since the objective of the current study was to compare techniques, not to provide a comprehensive analysis of the flight test, only a small portion of the available test data set is discussed.

The flight tests were conducted by an F-16 modified for flutter testing with a specialized excitation system. Of primary interest was the ability of the excitation system developed by Dynamic Engineering Inc., the DEI exciter,[12] to excite the primary vibration modes of the aircraft. The tests were conducted with two excitation techniques: 1) conventional flaperon excitation and 2) the DEI exciter. The first method involved the oscillatory deflection of the flaperons. The control surfaces were deflected using a linear sine sweep signal, and the position of the control surface was used as the excitation input signal in the data processing procedure. The second method using the DEI exciter involved the use of small, mechanically actuated vanes mounted on the outboard side of the midspan wing pylons. A slotted cylinder mounted on the vane was rotated to alter the aerodynamic loads on the vane, providing a periodic excitation force to the wing. Both linear and logarithmic sine sweep signals were used. A strain gauge was mounted on the connection between the vane and the pylon. This was used to measure the strain in the connection that should be

Table 3 Frequency domain estimates

		f_d	ζ	f_d	ζ	f_d	ζ
Flap	LSCE, forward	6.61	0.034	11.82	0.053	——	——
	LSCE, aft	6.60	0.033	11.78	0.043	——	——
	Curve fit, forward	6.60	0.048	11.82	0.031	——	——
	Curve fit, aft	6.63	0.042	11.74	0.037	——	——
DEI linear	LSCE, forward	6.85	0.019	12.40	0.045	17.43	0.029
	LSCE, aft	6.84	0.018	12.40	0.037	17.47	0.025
	Curve fit, forward	6.93	0.053	12.38	0.029	17.11	0.022
	Curve fit, aft	——	——	12.30	0.037	17.15	0.024
DEI log	LSCE, forward	——	——	——	——	——	——
	LSCE, aft	——	——	——	——	——	——
	Curve fit, forward	6.58	0.036	12.20	0.032	17.52	0.015
	Curve fit, aft	6.59	0.031	12.20	0.019	17.41	0.018

Table 4 Time-domain least-squares estimates

		f_d	ζ	f_d	ζ	f_d	ζ
Flap	SIMO, forward/aft	6.70	0.037	11.82	0.033	——	——
	SISO, forward	6.69	0.038	11.84	0.041	——	——
	SISO, aft	6.69	0.035	11.79	0.027	——	——
DEI linear	SIMO, forward/aft	——	——	12.35	0.018	17.50	0.017
	SISO, forward	——	——	12.30	0.018	17.51	0.018
	SISO, aft	——	——	12.45	0.022	17.49	0.019
DEI log	SIMO, forward/aft	——	——	——	——	17.40	0.006
	SISO, forward	——	——	——	——	17.34	0.006
	SISO, aft	——	——	——	——	17.47	0.007

proportional to the bending moment at the attachment between the vane and the wing.

The vibration of the wing was monitored with a number of accelerometers mounted at various locations. The data used in this study was restricted to single component accelerometers (normal or z plane) mounted at forward and aft positions on a launcher rail at the tip of the wing. These accelerometers provided the response signals for both the frequency and time domain methods. In each case, the excitation was provided with a sweep lasting approximately 25 s. The excitation and response data were collected at a sample rate of 200 Hz, resulting in time-domain data records containing approximately 5000 data points. For the frequency domain estimates a single channel of response output was used in each case so that the system was modeled as a single input/single output (SISO) process. The formulation of the time-domain data processing method allowed the output from both accelerometers to be considered either independently or simultaneously. Therefore the system could be characterized as either a SISO process or a single input/multiple output (SIMO) process. Additional details on the SIMO formulation of the TLS extension of the RBM can be found in Ref. 13.

The estimates using the frequency domain methods were generated by NASA personnel and though some of those results are included in this article, few details on these methods are provided. Two different frequency-domain-based techniques were used to develop modal estimates. The first method was developed by LMS International,[14] and involves the transformation of excitation and response data into the frequency domain where the cross power spectrum can be calculated using ensemble-averaged data. A linear system identification method, the least square complex exponential technique,[15] is then used to estimate the frequency and damping. The results developed using this method for two of the excitation techniques (flaperon and DEI linear) are labeled LSCE in Table 3. The table presents frequency in hertz and damping estimates for the first three modes at approximately 6.8, 11.8, and 17.4 Hz.

The second method was based on a similar technique except that the ensemble-averaged, autospectral density of the output signal was used with a curve-fit procedure provided by a MATLAB[16] toolbox routine. With the second technique, the analyses were conducted using block sizes of 2048 data points with 50% overlapping. The estimates generated using this approach are labeled Curve Fit in Table 3.

The results in Table 3 indicate the presence of three modes below 20 Hz. With the flaperon excitation, only the 6.7 and 11.8 Hz modes were identified with damping values ranging from approximately 0.03 to 0.05. A general estimate of the uncertainty in the damping estimates can be made using these results and there is an indication of considerable variation depending on response transducer, excitation type, and data processing method. The LSCE estimates were not developed for the DEI Log excitation.

Before the time domain identification methods were used to process the flight-test data, some simple signal processing techniques were applied. As described earlier, for each test case, approximately 25 s of test data was available at a sample frequency of 200 Hz. At this sample rate, with appropriate low-pass filtering, one can hope to identify modes in a frequency range from 0 to 100 Hz (i.e., the Nyquist frequency). It was known that the wing modes were in a range from 0 to 20 Hz, so that at this sampling rate all of the poles would be located in a small region of the first quadrant of the z domain. Therefore the data were digitally, low-pass filtered at 20 Hz and every fifth point selected such that the effective sample frequency was 40 Hz. A 5600-point, 25-s signal was reduced to a 1120-point, 25-s time record. Both the excitation and response signals were filtered with the same fourth-order, Butterworth digital filter and the resulting filtered signals were used for all of the time-domain estimation efforts. Each signal was then normalized by scaling to an rms value of 1.0.

Estimates of the frequency and damping of the modes under 20 Hz were generated using the time-domain identification techniques outlined earlier using both LS and TLS criteria. For all of the time-domain estimates the model order was chosen to be 50 and the order of the signal subspace was selected to be 6 to correspond to the three modes that were expected in the frequency range of interest. Unlike the LS formulation of RBM, an estimate of the size of the signal subspace is necessary for the TLS implementation and this limits complete automation of the approach. The frequency (Hz) and damping estimates for the LS approach are presented in Table 4 and the TLS results in Table 5. For each type of excitation, there are

Table 5 Time-domain total least-squares estimates

		f_d	ζ	f_d	ζ	f_d	ζ
Flap	SIMO, forward/aft	6.69	0.047	11.88	0.052	—	—
	SISO, forward	6.87	0.058	—	—	—	—
	SISO, aft	6.72	0.051	—	—	—	—
DEI linear	SIMO, forward/aft	6.97	0.014	12.37	0.028	17.51	0.025
	SISO, forward	—	—	12.25	0.015	17.55	0.025
	SISO, aft	6.90	0.011	12.45	0.031	17.48	0.025
DEI log	SIMO, forward/aft	6.17	0.010	12.26	0.014	17.42	0.019
	SISO, forward	6.83	0.009	12.25	0.014	17.31	0.021
	SISO, aft	6.70	0.008	12.26	0.009	17.50	0.017

three estimates. The first is based on a SIMO, ARMA model, and the second two are based on SISO ARMA models, and therefore, use only one of the two available accelerometer response channels. This allows for direct comparison with the SISO frequency domain results.

For all of the time domain presented, the backward time technique was used to provide an automated approach for mode discrimination since order overspecification was used in each case. The LS results shown in Table 4 illustrate some of the problems associated with the noise-biased results. For both the SIMO and SISO results, not all of the modes were automatically identified. The flaperon excitation allowed for the identification of the first two modes where the DEI excitation method provided the second and third modes. On average the damping estimates were lower than those achieved with the frequency domain techniques. The TLS results in Table 5 indicate that most of the modes were identified, particularly for the DEI excitation. The TLS damping values are consistently greater than those achieved using the LS criteria. On occasion the TLS approach resulted in additional modes identified as being present within the unit circle. Whether they are caused by extraneous poles or actual system modes, either local or global that were not identified using the other methods, cannot be determined from this limited data set. Truly automated mode identification and sorting is still an issue of concern for future research.

It was determined subsequent to the tests that the DEI exciters were not placed in the best positions to effectively excite the wing when these results were compared with other flight tests using the DEI exciter. This made the data processing for both the frequency and time-domain techniques more difficult. One unfortunate characteristic of this flight-test data is that there were no closely spaced modes that could be used to compare the use of frequency and time-domain techniques.

Although it is difficult to make any conclusive statements about the results obtained with this data, it is possible to make some observations about the data processing techniques. There was a time savings involved in the use of the time-domain methods. The frequency domain estimates each took from 2 to 5 min to generate. In addition, there was a significant amount of user interaction in developing these estimates. The time-domain estimates were obtained in 30 s to 2 min on the same workstation. The SIMO, ARMA models took about twice as long as the SISO models. Although some time savings could be achieved in both of these methods, the relative speeds would likely remain approximately the same. Although the time difference may not seem to be significant, if the estimates are required during a flight test and at a number of flight test conditions, the additional time can have an impact.

Overall, the time-domain models seem to provide effective estimates of both frequency and damping. Two exceptions are noted: first, because the flaperon did not excite the 17-Hz mode, neither the frequency domain nor the time-domain methods were able to consistently identify this mode; second, inconsistent estimates were generated for the 6-Hz mode in the two cases when the DEI exciter was used as the excitation source. An examination of the coherence function generated with the DEI strain gauge and the output accelerometers, revealed that although the 6-Hz mode was being excited slightly by some mechanism, the DEI excitation was not primarily responsible for the response.

Some final comments are relevant to this study.

1) Some averaging of the results may decrease the error in the results because of precision. Overlapping blocks of time series data could be used to develop multiple frequency and damping estimates and average parameter values determined.

2) The data reduction process was repeated for overspecified levels above 20 and there did not appear to be a significant increase in the accuracy of the results. Lowering the overspecification level would decrease the computational time and may allow for comparable accuracy.

3) Other than filtering, resampling, and normalization, no other modifications were made to the time-domain data. This means that user interaction is limited and it helps to expedite the data reduction process.

Conclusions

Both numerical simulation and flight flutter test data were used to evaluate the application of the total least-squares criteria to the parameter identification for ARMA models of linear dynamic systems. The results support earlier efforts that indicate that there is significant bias in parameter estimates, in particular modal damping estimates, when least-squares methods are used. The use of the total least-squares criteria that recognizes the influence of uncorrelated noise in the experimental measurement was shown to have a positive effect on the parameter bias. The use of the total least-squares criteria along with averaging of the parameter estimates appeared to reduce both bias and precision errors and reduce the overall uncertainty in the parameter estimates.

The extension of the RBM approach by using the total least-squares criteria has shown to improve the parameter estimates using time-domain methods when applied to the flight flutter test data. The pole sorting is improved because of the reduced bias, and damping estimates appear to be comparable to frequency domain approaches. The amount of time and user interaction is reduced when the time domain techniques are compared with current frequency domain approaches. With additional experience the time domain approaches may provide an attractive and economical alternative.

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